ASYMPTOTIC EVALUATION OF RESONANT FREQUENCIES FOR TWO COUPLED CIRCULAR MICROSTRIP DISK RESONATORS

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Abstract

For two coupled circular microstrip disk resonators in tne limit of small substrate thickness, a matched asymptotic expansion approach is used to derive asymptotic formulae for the resonant frequencies. The mode coupling effects are clearly demonstrated.

Introduction

The resonance in single-element microstrip antennas has been extensively studied in the past few years[2-8]. On the other hand, less efforts have been directed to the analysis of the coupling effects between several elements. The coupling between two circular microstrip disk resonators has been studied using an electrostatic approach[9]. More recently, the resonance of the two circular microstrip structure has been developed using a full wave analysis[1,11], and the mutual coupling of rectangular microstrip antennas was studied using a moment method approach[10].

In this paper, the matched asymptotic expansion approach is used to asymptotically evaluate the resonant frequencies of the two circular disk resonator. This method was used in Refs. 12 and 13 to develop an asymptotic formula for the resonant frequencies of a circular and an annular ring microstrip antenna.

In carrying out the asymptotic expansions we will only keep track of terms of the order of $\delta = d/a$ where d is the substrate thickness and a is the radius of the two circular disks.

Formulation

Figure 1 shows the geometry of the problem. Two local cylindrical coordinate systems are defined by (ρ_j, ϕ_j, z), $j = 1, 2$ referred to the centers O_j of disks D_j. The space around the open resonators is divided into five regions, two interior regions, two edge regions and an exterior region.

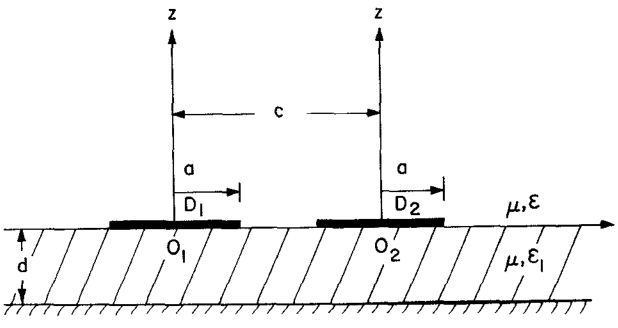

Figure 1

Geometrical configuration of the two coupled circular microstrip disk resonators.

A. The Interior Region

Making use of the coordinate transformation $\rho_j = \rho_j$, $\phi_j = \phi_j$, and $z = a\delta Z$ and taking the limit of $\delta \to 0$, we write

$$h^{(j)}_{1\phi} \sim \frac{ik_1}{\omega\mu} \sum_n e^{in\phi_j} E_n^{(j)} J_n'(k_1\rho_j)$$

where $E_n^{(j)}$ is an unknown. The edge expansion of this interior solution is obtained by setting $\rho_j = a(1 + \delta X_j)$ and $\delta \to 0$.

$$h^{(j)}_{1\phi} \sim \frac{ik_1}{\omega\mu} \Bigg[e^{in\phi_j} E_n^{(j)} \Big\{ J_n'(k_1 a) + \delta X_j \frac{(n^2 - k_1^2 a^2)}{k_1 a} J_n(k_1 a) \Big\} + e^{-in\phi_j} E_{-n}^{(j)} (-1)^n \Big\{ J_n'(k_1 a) + \delta X_j \frac{(n^2 - k_1^2 a^2)}{k_1 a} J_n(k_1 a) \Big\} \Bigg] .$$

B. The Edge Region

This region is emphasized by the following coordinate transformation $\rho_j = a(1 + \delta X_j)$, $\phi_j = \phi_j$, and $z = a\delta Z$. From Ref. 12, and in the limit $\delta \to 0$, we get

$$h^{(j)}_{1\phi} \sim e^{in\phi_j} \left\{ i \frac{B_n^{(j)}}{n} (k_1^2 a^2 - n^2)\, \delta X_j + C_n^{(j)} \right\} + e^{-in\phi_j} \left\{ - \frac{iB_{-n}^{(j)}}{n} (k_1^2 a^2 - n^2)\, \delta X_j + C_{-n}^{(j)} \right\}$$

for the interior expansion of the edge solution. Whereas in the limit $(X_j^2 + Z^2)^{1/2} \to \infty$, we get

$$h^{(j)}_{o\phi} \sim e^{in\phi_j} \left\{ i \frac{B_n^{(j)}}{n} \frac{\delta}{\pi} \{(k_o^2 a^2 - n^2) \ln(X_j^2 + Z^2)^{1/2} + S_n\} + C_n^{(j)} \right\} + e^{-in\phi_j} \left\{ - i \frac{B_{-n}^{(j)}}{n} \frac{\delta}{\pi} \{(k_o^2 a^2 - n^2) \ln(X_j^2 + Z^2)^{1/2} + S_n\} + C_{-n}^{(j)} \right\}$$

for the exterior expansion of the edge solution where

$$S_n = k_o^2 a^2 \frac{A}{2} - \frac{n^2}{2} (\ln(\pi) + 1)$$

$$A = -2\varepsilon_r \sum_{m=1}^{\infty} \left(\frac{1 - \varepsilon_r}{1 + \varepsilon_r}\right)^m \ln(m) + \varepsilon_r \ln(\pi) + (\varepsilon_r - 1) \ln(2) + 1$$

$\varepsilon_r = \varepsilon_1/\varepsilon_o$ and $B_n^{(j)}$, $C_n^{(j)}$ are unknowns to be determined through the asymptotic matching.

C. The Exterior Region

In the small δ limit, only modal fields with no z-variations inside the substrate will be excited and since all TE modes have z-variation, none of them will be excited, the currents on the disks are thus expanded in terms of the TM modes. Substituting $\rho_j = a(1 + \delta X_j)$, $z = a\delta Z$ and letting $\delta \rightarrow 0$, we get the edge expansion of the exterior solution[1,11,12]

$$h_{o\phi}^{(j)} \sim i\delta a^2 \frac{J_n(\beta_{nm})}{\beta_{nm}} \Bigg[e^{in\phi_j} \Big\{ A_n^{(j)} \Big[\eta + \frac{i}{\pi a^2} (k_o^2 a^2 - n^2) \ln(X_j^2 + Z^2)^{1/2} \Big] + A_n^{(j')} \eta_c + A_{-n}^{(j')} \bar{\eta}_c \Big\}$$
$$+ (-1)^n e^{-in\phi_j} \Big\{ A_{-n}^{(j)} \Big[\eta + \frac{i}{\pi a^2} (k_o^2 a^2 - n^2) \ln(X_j^2 + Z^2)^{1/2} \Big] + A_n^{(j')} \bar{\eta}_c + A_{-n}^{(j')} \eta_c \Big\} \Bigg]$$

where $j' = 1$ if $j = 2$ and $j' = 2$ if $j = 1$

$$\eta = \frac{i}{\pi a^2} \Bigg[(k_o^2 a^2 - n^2) \Big\{ \ln(\delta/8) + 2 \sum_{k=1}^{n} \frac{1}{2k - 1} \Big\} - \frac{2k_o^2 a^2}{4n^2 - 1} + (-1)^n \int_0^{\pi/2} d\psi \cos(2n\psi) \frac{\exp(i2k_o a \cos\psi) - 1}{\cos\psi} (n^2 + k_o^2 a^2 \cos 2\psi) \Bigg]$$

$$\eta_c = \int_0^{\infty} dk_\rho k_\rho J_o(k_\rho c) \left[-\frac{n^2 k_z}{k_\rho^2 a} J_n^2(k_\rho a) + \frac{k_o^2 a}{k_z} J_n'^2(k_\rho a) \right]$$

$$\bar{\eta}_c = \int_0^{\infty} dk_\rho k_\rho J_{2n}(k_\rho c) \left[-\frac{n^2 k_z}{k_\rho^2 a} J_n^2(k_\rho a) + \frac{k_o^2 a}{k_z} J_n'^2(k_\rho a) \right].$$

By matching the different asymptotic solutions we obtain the following eigenequations for the possible excited modes in the coupled structure

$$k_1 a \sim \beta_{nm} \left[1 - \frac{\delta}{\pi(\beta_{nm}^2 - n^2)} \{(S_n + i\pi a^2[\eta + C_1 \eta_c - C_2(-1)^n \bar{\eta}_c]\} \right]$$

where $C_1 = 1$ and $C_2 = 1$ for the odd-symmetric modes which are modes that vary as $\sin(n\phi_1)$ on disk D_1 and $\sin(n\phi_2)$ on disk D_2. $C_1 = -1$ and $C_2 = -1$ for the odd-antisymmetric modes which are those that vary as $\sin(n\phi_1)$ on D_1 and $-\sin(n\phi_2)$ on D_2. $C_1 = 1$ and $C_2 = -1$ for the even-symmetric modes which are varying as $\cos(n\phi_1)$ on D_1 and $\cos(n\phi_2)$ on D_2. Finally, $C_1 = -1$ and $C_2 = 1$ for the even-antisymmetric modes which vary as $\cos(n\phi_1)$ on D_1 and $-\cos(n\phi_2)$ on D_2 where $J_n'(\beta_{nm}) = 0$.

Results and Conclusions

In this paper the two coupled-circular-disk structure is analyzed using the matched asymptotic expansion approach. The asymptotic formula of the resonant frequencies is shown to account for the coupling effects between the two disks. It is seen that the structure can support four different resonant modes: odd-symmetric (os), even-symmetric (es), odd-antisymmetric (oa), and even-antisymmetric (ea).

Figures 2a and 3a show the real part of the resonant frequency of the four different TM_{11} resonant modes, whereas Figs. 2b and 3b show the imaginary part as a function of the thickness of the substrate d/a for a separation of c = 2.05 a between the two disks.

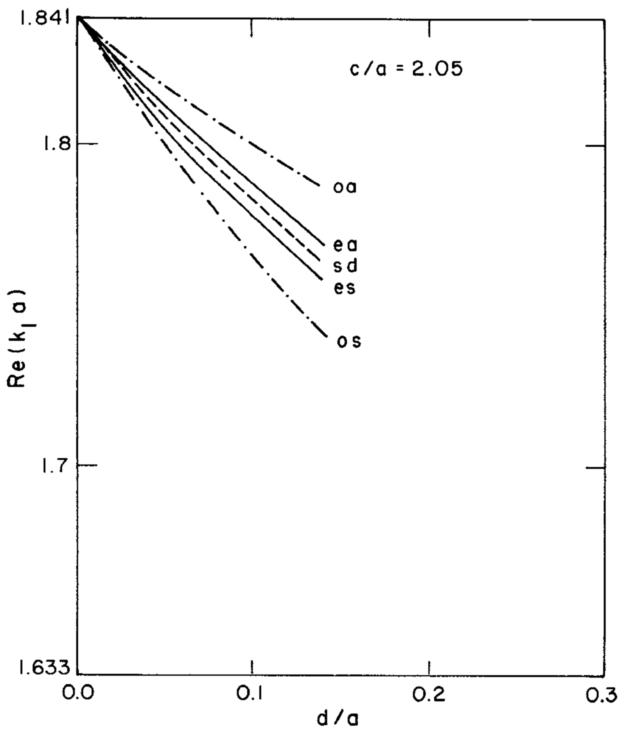

Figure 2a

Real part of the resonant frequencies of the different modes of the TM_{11}, as a function of d/a using the matched asymptotic expansion approach (MA), C = 2.05a, $\varepsilon_1 = 2.65\varepsilon$.

Figures 2a and 2b are the results of the asymptotic formula for the resonant frequencies obtained using the matched asymptotic expansion approach whereas Figs. 3a and 3b are those obtained using a perturbational formula[1] and are included in this paper for comparison.

Comparing the results obtained from the matched asymptotic expansion approach to those obtained from

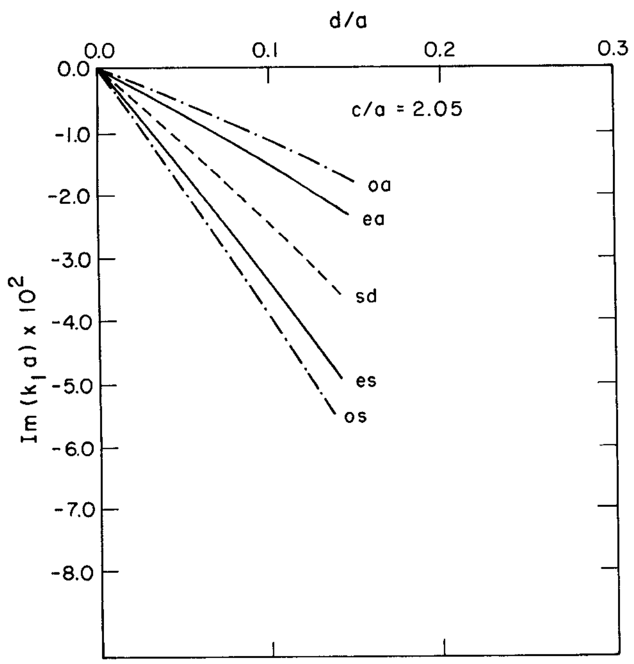

Figure 2b

Imaginary part of the resonant frequencies of the TM_{11} mode using (MA). Same parameters as in Fig. 2a.

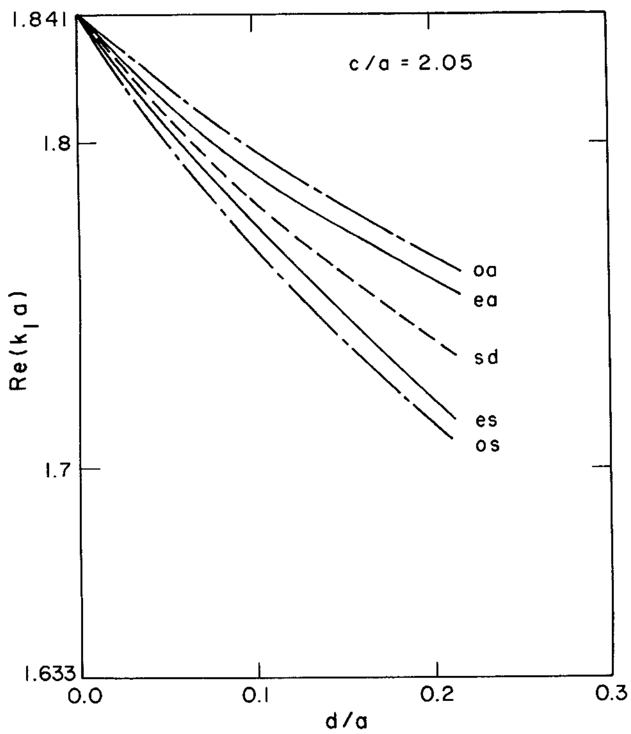

Figure 3a

Real part of the resonant frequencies of the different modes of the TM_{11}, as a function of d/a using the perturbational approach (PA), $C = 2.05a$, $\varepsilon_1 = 2.65\varepsilon$.

the perturbation approach, it is clear that the two methods agree quite satisfactorily for values of d/a ≤ 0.1. On the same figures the resonant frequency of the single disk (sd) is plotted. It is clear from Figs. 2b or 3b that the excitation of the odd-symmetric TM_{11} mode makes the structure act as a better antenna than the single disk whereas the odd-symmetric mode makes it a better resonator.

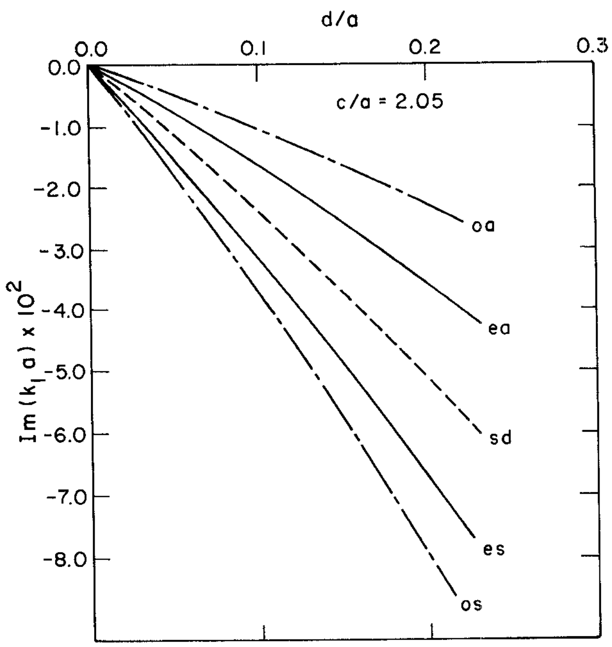

Figure 3b

Imaginary part of the resonant frequencies of the TM_{11} mode using (PA). Same parameters as in Fig. 3a.

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